

# Impact of the leads on the bound states in the continuum in double quantum dots

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## Abstract

Bound states in the continuum (BICs) are truly localized states lying in the continuum energy spectrum of a quantum system. They were postulated by von Neumann and Wigner already in 1929 [1] and the advent of nanofabrication techniques has made it possible to devise and fabricate quantum devices to experimentally validate their existence. Capasso *et al.* measured the absorption spectrum at low temperature of a GaInAs quantum well with Bragg reflector barriers produced by a AlInAs/GaInAs superlattice [2]. A narrow line in the absorption spectrum was attributed to electron excitations from the ground state of the quantum well to a localized level well above the AlInAs band edge. Nevertheless, this state cannot be regarded as a true BIC but a bound state above the barrier since it is a defect mode residing in the minigap of the superlattice, as pointed out by Plotnik *et al.* [3].

Electronic transport in mesoscopic and nanoscopic systems can be also influenced by the occurrence of BICs [4]. González *et al.* have demonstrated that quantum dots based on semi-conductor materials and graphene can support BICs [5]. Furthermore, Dutta and Roy have shown that BICs may arise in heterogeneous nanostructures by engineering the spatial dependence of the effective mass of carriers [6]. Recently, González-Santander *et al.* extended the notion of BIC to the domain of time-dependent potentials [7]. To this end, they considered a quantum ring connected to two leads. An AC side-gate voltage controlled the interference pattern of the electrons passing through the system. The transmission probability displayed Fano antiresonances when the Fermi energy matches the driving frequency, signaling the presence of BICs. All these features stimulate the interest of BICs to develop new applications in nanoelectronics.

In this work we consider two quantum dots, tunnel-coupled to left (L) and right (R) leads, as shown in Fig. 1. The energy levels of the dots,  $\varepsilon_i$ , can be shifted by two side electrodes. In addition, electrons of each dot interact with a local vibrational mode with frequency  $\omega_0$ . The Hamiltonian describing the system is (we set  $\hbar = 1$ )

$$H = \sum_{\ell k} \varepsilon_{\ell k} c_{\ell k}^\dagger c_{\ell k} + \sum_i \left\{ \omega_0 a_i^\dagger a_i + \left[ \varepsilon_i + \lambda (a_i^\dagger + a_i) \right] d_i^\dagger d_i \right\} + \sum_{\ell k} \left[ V_{\ell k i} c_{\ell k}^\dagger d_i + \text{H.c.} \right]$$

where  $c_{\ell k}$  destroys an electron with wave number  $k$  and energy  $\varepsilon_{\ell k}$  in the lead  $\ell$  (L or R),  $d_i$  is the destruction operator in the  $i$ th dot and  $a_i$  a local vibrational mode at the dots. Here  $V_{\ell k i}$  is the tunneling matrix element between the dots and the leads and  $\lambda$  is the electron-phonon coupling constant. We closely follow Ref. [8] and compute the electric current using the nonequilibrium Keldysh formalism.

$$I = \frac{e}{2\hbar} \int d\omega \left\{ \text{Tr} \left[ \left( f_L(\omega) \Gamma^L - f_R(\omega) \Gamma^R \right) A(\omega) + \text{Tr} \left[ \left( \Gamma^L - \Gamma^R \right) i G^<(\omega) \right] \right] \right\}$$

where  $f_\ell$  is the Fermi distribution function,  $A(\omega) = i(G^>(\omega) - G^<(\omega))$  is the spectral function and the

Green's functions are defined in the standard form. The matrices  $\Gamma^\ell$  describes the tunneling coupling between the dots and the leads

$$\Gamma^L = \begin{pmatrix} 1 & \sqrt{\alpha} \\ \sqrt{\alpha} & \alpha \end{pmatrix} \gamma_0 \quad \Gamma^R = \xi \begin{pmatrix} \alpha & \sqrt{\alpha} \\ \sqrt{\alpha} & 1 \end{pmatrix} \gamma_0$$

Here  $\gamma_0$  is a constant,  $\alpha$  describes the difference in the coupling of the electrodes to different dots and  $\xi$  stands for the asymmetry in the coupling of the QDs to the left and right leads.

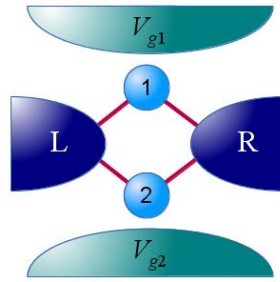
In our calculations we take  $\omega_0$  as the unit of energy and work at  $T = 0$ . In addition, we consider  $\gamma_0 = 0.02$ ,  $\lambda = 0.1$ ,  $\alpha = 0.4$  and  $\xi = 1$ , and calculated the spectral function  $A(\omega)$ . Figure 2 shows the results for asymmetric leads, namely taking  $\Gamma^L = \Gamma^R$ , and symmetric leads. In both cases the results for  $\varepsilon_1 = \varepsilon_2$  and  $\varepsilon_1 = -\varepsilon_2 = -0.02$  are shown. When the leads are asymmetric, the spectral function is rather insensitive to variations of the energy levels of the dots. On the contrary, the spectral function when the lead are

symmetric reveals the existence of the BICs as well defined Fano resonances, as expected in this system [7].

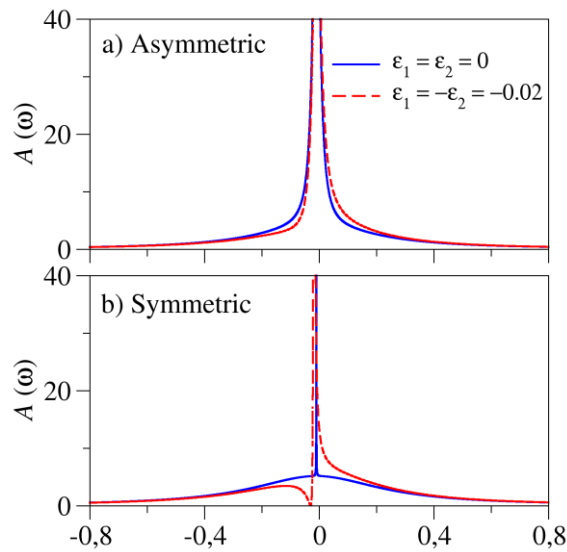
## References

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## Figures



**Figure 1.** Schematic view of the device. Two quantum dots, labelled 1 and 2, tunnel-coupled to left (L) and right (R) leads. Two side electrodes with voltage  $V_{g1}$  and  $V_{g2}$  shift the energy levels of the dots.



**Figure 2.** Spectral function for a) asymmetric and b) symmetric leads. Parameters are given in the main text.